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Abstract

Ziebart [1985] demonstrates that, given a particular level of confidence, the magnitude of an unexpected return is linked to the standard error of the beta coefficient used to compute the abnormal return. This paper provides evidence linking the reliability of the beta estimate to trading activity using the subordinated stochastic process hypothesis. The associations between the level of trading activity and both the standard error of the beta estimate and the coefficient of determination for the market model regression are assessed on both the individual security level and the portfolio level.

1.0 Introduction

The market model is a common component in many market based empirical research studies in accounting and finance. It is used to separate security returns into systematic and unsystematic components. In most applications, the regression equation fit and the reliability of the regression coefficient (beta) vary greatly from one security/portfolio to another. For the most part this situation is assumed to be inconsequential since unsystematic risk can be diversified away and most analyses use large samples which could randomize the effect. However, Ziebart [1985] demonstrates that, given a particular level of confidence, the magnitude of the unexpected return is linked to the standard error of the beta coefficient. He empirically demonstrates this link using both individual securities and portfolios.

Given this link between abnormal returns and the reliability of the regression coefficient, this paper provides evidence regarding an explanation of the varying degrees of regression model fit and regression coefficient reliability. Based on the subordinated stochastic return process hypothesis, the relationship between the trading activity of a security and the empirical characteristics of the estimated market model is investigated. Specifically, the associations between the level of trading activity and both the standard error of the beta estimate and the coefficient of determination for the regression are assessed for both individual securities and portfolios. The results support the linkage between beta reliability and trading

activity. However, this paper only addresses the link between trading activity and reliability of the regression coefficient within the market setting of the New York Stock Exchange. As such, the thin trading phenomenon which normally is deemed to exist across varous market structures (such as NYSE, ASE, and OTC) is not investigated. The thin trading effect does impact the estimation of beta but it is related to the market structure and size of the market rather than the subordinated stochastic process hypothesis. In addition, monthly returns are used for the analysis in order to study the effects of the subordinated stochastic process hypothesis without potential contamination due to nonsynchronous trading.

The reliability of the systematic risk coefficient may be important in both investment and research contexts. If one desires a target level of systematic risk for an investment, the uncertainty regarding the beta estimate affects the reliability one can place on the expected performance of the security or portfolio relative to the market. Estimation risk and its effects on optimal portfolio choice have been studied by Chen and Brown [1983] and Bawa, Brown, and Klein [1979]. One common control strategy in empirical research is to match betas of individual secuities or portfolios. While this may attempt to control for the level of systematic risk, the problem regarding the certainty that can be placed in the estimated coefficient still exists.

A security with a beta estimate of 1.00 and a standard error of the estimate for beta of .20 is not the same as a security with a beta estimate of 1.00 with a standard error of .05. The second

security has a tighter distribution around its market model equation and a comparison of abnormal returns (risk adjusted) between the two securities may indicate a significant difference which may be due to the difference in the fit of the two market models [Ziebart 1985]. This problem may also occur when one formulates iso-beta portfolios; the portfolios have beta estimates of 1.00 but varying levels of beta reliability and market model fit. Differences in abnormal returns at the portfolio level may be linked to the reliability of beta [Ziebart 1985].

The next section of this paper provides a link between trading activity and beta reliability. Empirical evidence of the relationship on the individual security level is provided in the third section while the fourth section empirically investigates the relationship on a portfolio level. A summary of the results and the conclusions are provided in the final section.

2.0 Link Between Trading Activity, Beta Reliability, and Market Model Fit

Previous empirical analyses by Kendall [1953], Mandelbrot and Taylor [1967] and others find return distributions measured over calendar periods of time yield a higher frequency of observations near the mean and in the tails than would be expected for a normal distribution. Distributions of returns exhibit leptokurtosis; they are generally fat-tailed and peaked. One explanation of this phenomena is put forth by Mandelbrot and Taylor [1967], Granger and Morganstern [1970], Clark [1973], and Westerfield [1977]. They hypothesize that returns are generated by a subordinated stochastic process. This implies that the rate of evolution in

the return generation process is not a function of calendar time but is a function of another underlying process within each calendar time period.

The return for a security over a period of calendar time reflects the accumulation of new information occurring during that time period (Westerfield [1977]). If the number of new information bytes is itself a random variable, then the return for that calendar period of time can be depicted as the result of a subordinated stochastic process. The return is the result of a sum of a random number of news events which occur during the calendar period of time. Mandelbrot and Taylor [1967], Granger and Morganstern [1970], and Clark [1973] introduce the notion of transaction time in the subordinated stochastic model. Since the number of new information bytes to the market should be manifested in the number of transactions ocurring during a calendar period of time, the return generation process can be depicted as being directed by the transaction process. Using trading activity or volume as a measure of transactions implies that the return generation process is a stochastic function of trading volume.

Calendar time is the common index employed when stock returns are measured. Let us depict returns as:

$$r_{it},\ r_{it+1},\ r_{it+2},\ \ldots\ r_{it+n}$$

(where i denotes firm and t denotes a calendar period of time). Each return, r_{it} is the realization of a stochastic process for a particular calendar period of time, t. A subordinated stochastic return process assumes that within the calendar period of time t another stochastic process occurs. This stochastic process is

trading activity.

The return for firm i over period t can be expressed as:

$$r_{it} = r_{i(V\{t\})} \tag{1}$$

(where (V{t}), the directing process, is the level of trading activity). Robbins [1948] and Clark [1973] demonstrate the following:

if r_{it} can be drawn from a distribution with mean 0 and finite variance σ^2 and the changes in V{t} can be drawn from a positive distribution with mean μ, independent of the changes in r_t, then the subordinated stochastic process $r_{i(V\{t\})}$ has stationary independent changes with mean 0 and variance $\mu\sigma^2$.

The variance of $r_{i(V\{t\})}$ conditional upon V{t} is:

$$\text{var} \left[(r_{i(V(t))}) \mid (V(t)) \right] = V_t \sigma^2 \tag{2}$$

Recall that V_t is the directing process, trading activity, and σ^2 is the variance of the return realization. Clark [1973] provides additional details and a proof of this relationship.

Let X and Y represent two securities and, that in the absense of any informational shocks, their return generation processes each generate a return disrtibution with mean 0 and finite variance σ^2. Trading occurs as new information reaches the market. Given different types and numbers of news events for the two securities differential levels of trading result and the variances of the return distributions for X and Y are (from (2)):

$$\text{var} \left[(R_{X(V(t))}) \mid (V(t)) \right] = V_X \sigma^2 \tag{3}$$

$$\operatorname{var}\left[(R_{Y(V(t))}) \mid (V(t))\right] = V_Y \sigma^2 \quad (4)$$

The variance of the return distribution for time t is positively linked to the level of trading that occurs during the calendar time period. Across firms, for time t, the variances of the return distributions differ depending on the trading level which occurs during the period of observation.

In applications of the market model, the return variance is a function of beta, the variance of the market return, and the variance of the unsystematic return (or error variance of the regression):

$$\sigma^2 R_i = \hat{\beta}^2 \ \sigma^2 R_m \ \ \sigma^2 e_i \quad (5)$$

The beta coefficient estimate is the covariance of the return on the individual security with the market return divided by the variance of the market return:

$$\hat{\beta} = \frac{\sigma_{R_i R_m}}{\sigma^2_{R_m}} \quad (6)$$

Given differential levels of trading for securities X and Y such that the trading volume of security X during time t is greater than the trading volume of security Y during time t, the variance of the return distribution for X should exceed the variance of the return distribution for Y:

$$\sigma^2_{R_X} > \sigma^2_{R_Y} \quad (7)$$

Therefore, through application of the market model, the sum of the

variance components for security X exceed the sum of the variance components for security Y:

$$\hat{\beta}^2_X \sigma^2_{R_m} + \sigma^2_{e_X} > \beta^2_Y \sigma^2_{R_m} + \sigma^2_{e_Y} \tag{8}$$

Let us assume that the two securities X and Y possess the same systematic risk such that the beta of X equals the beta of Y:

$$\beta_X = \beta_Y \tag{9}$$

Since the variance of the market return is the same across securities and since the systematic risk is the same (by assumption) for the two securities, the nonsystematic risk or error variance for the market model regression is larger for X than Y:

$$\sigma^2_{e_X} > \sigma^2_{e_Y} \tag{10}$$

Given equal betas (by assumption) and since the variance of the market return is the same for both securities at the same point in time, the error variance term reflects the differential levels of trading. Since the differential levels of trading are the result of varying amounts of information bytes for the two securities, the variance of the error term reflects the differential amounts of information for the two securities.

Equation (10) implies that the market model fit, the coefficient of determination, is less for X than for Y. Accordingly, the standard error of the beta estimate for security X is greater than the standard error of the beta estimate for security Y. This follows since the betas are equal, the variance

of the market return is equal accross the two securities, the variance of the return of security X is greater than the variance of the return of security Y, and the standard error of the beta estimate conditional upon the beta estimate is computed as:

$$SE(\hat{\beta}) = \left[\frac{\hat{\sigma}^2_{R_i} - \hat{\beta}^2_i \, \sigma^2_{R_m}}{\Sigma \, (R_{m_t} - \bar{R}_m)^2} \right]^{1/2} \tag{11}$$

Recall from (2) that the variance of the return distribution for X is larger than the variance of the return distribution for security Y because the number of transactions (trading volume) is greater for security X than security Y. Assuming the return generation process is a subordinated stochastic process based on trading activity, a positive relationship is expected between trading volume and the standard error of the beta estimate, and a negative relationship is expected between the coefficient of determination (R^2) and trading volume.

3.0 Empirical Evidence Using Individual Securities

In order to provide empirical insight into the role which trading activity plays in the representativeness of the estimated market model the hypothesized relationships are studied on the individual firm level. A random sample of 213 manufacturing firms on the New York Stock Exchange that met the following criteria were chosen.

1. Each firm must have a complete history of return data on

the CRSP monthly return data base for the January 1, 1975 through December 31, 1979 time period.

2. Each firm must have a complete history of trading activity on the ISL Daily Stock Record.

Monthly observations were chosen for study in order to control for the effects of nonsynchronous trading which are more acute when weekly or daily observations are used.

Using ordinary least squares regression, the market model was estimated for each security by regressing the individual security returns on the market returns for the sixty month period. The coefficient of determination, R^2, and the standard error of the beta estimate, $SE(\hat{\beta})$, were calculated for each individual firm. Two measures of trading activity for the five year period were developed. A monthly mean trading volume measure was computed after adjustments were made for any stock splits, stock dividends, or new stock issues which occurred during the sixty month period. As a second measure the mean monthly shares traded was divided by the mean monthly shares outstanding to control for firm size effects. Table 1 provides the lower left triangle of the correlation matrix for the beta estimates, the standard errors of the beta estimates, the coefficients of determination, and the two measures of trading activity.

[INSERT TABLE 1]

As Fama [1976] points out, larger firms tend to have betas which are substantially less than one and usually the larger the beta the larger the variance in the return distribution. This results in a positive relationship between the estimated beta and

the standard error of the beta estimate. The results in Table 1 indicate a highly significant correlation between the two of .517. However, since the variance in the return distribution is a function of trading activity and since larger firms tend to have higher levels of trading and lower betas one would expect to observe negative relationships between trading volume and beta, trading volume and the standard error, and trading volume and the coefficient of determination. This is confirmed by the results in Table 1. The correlation between the standard error of the beta estimate and the mean trading activity is only -.083 while the correlation between the market model fit and mean trading activity is -.113. These results imply that the mean trading level metric may not be a good surrogate for the number of information bytes being recieved by the market for each security since the results are counter to what is expected via the subordinated stochastic process hypothesis.

Evidence in favor of the predicted relationships of the subordinated stochastic process is found when the level of trading is corrected for firm size by the number of shares outstanding. The correlation between the standard error of the beta estimate and mean relative trading activity is .339 while the correlation between the coefficient of determination and the mean relative trading activity is -.178. Both of these correlations are highly significant. These results provide evidence which supports on the individual firm level the notion that beta reliability and market model fit are linked to trading activity.

4.0 Empirical Analysis Using Portfolios

A common presumption in portfolio theory is that unsystematic risk can be diversified away and that market models estimated for portfolios should have higher coefficients of determination and lower standard errors for the beta estimates. Two subsamples of the 213 firms studied at the individual security level were chosen to test the effect of trading activity on market models estimated for portfolios. One subsample consisted of 40 firms which traded at least an average of 1,000,000 shares per month and had a high relative average also. The second group consisted of 40 firms which traded less than an average of 100,000 shares per month and a low relative average. Sixty portfolios of 15 randomly chosen securities were constructed from each group. This resulted in 60 portfolios comprised of high trading activity securities and 60 portfolios of low trading activity securities.

Using the beta estimates from the individual firm analysis, the 15 securities in each portfolio were weighted such that the portfolio had a beta of 1.00. Weighting was accomplished by segregating the securities of each portfolio into high and low beta segments, determining the average beta for each of the two segments, and then calculating the weight needed to drive the overall beta of the portfolio to one.

Table 2 provides the standard errors of the beta estimates and the coefficients of determination for the estimated market models of the high and low trading groups of portfolios.

[INSERT TABLE 2]

For the high trading group of portfolios the mean standard error for the beta estimate is .0855 with a variance of .0078. The low

trading activity group of portfolios has a mean standard error for the beta estimate of .0495 with a variance of .00003. A parametric test of a difference in means has a z score of 3.154 and using a one-tailed test the null hypothesis of no difference is rejected at a significance level of .00085.

The mean coefficient of determination for the high trading group of portfolios is .75 with a variance of .0040. The low trading group has a mean coefficient of determination of .87 with a variance of .0007. The mean coefficients of determination are significantly different at the .00001 level when a one-tailed test is applied.

The Wilcoxon rank-sum test for independent samples was conducted as an alternative to the parametric tests and the results were consistent with the parametric test results. These results also provide evidence in support of the hypothesized relationships between trading activity and attributes of the estimated market model on the portfolio. By forming iso-beta portfolios the effects of size and beta level were controlled.

5.0 Conclusions

This paper demonstrates that certain relationships should exist between trading activity and specific attributes regarding the fit of the estimated market model for a portfolio or security. Specifically, this analysis predicted that, given the return generation process of securities is subordinate to trading activity, (1) actively traded securities or portfolios will have market models exhibiting lower coefficients of determination than

less actively traded securities or portfolios and (2) the reliability of the beta estimate for actively traded securities or portfolios will be less than the reliability of the beta estimate for less actively traded securities or portfolios. The empirical results on both the individual security level and the portfolio level were in conformance with these predictions when trading activity is standardized for the number of shares outstanding. This result implies that raw trading activity is not a good surrogate for the number of information bytes being received by the market. In addition, the empirical results provide cross-sectional evidence which supports the subordinated stochastic process hypothesis.

Table 1: Lower Left Triangle of Correlation Matrix for Beta Estimates, Standard Errors of the Beta Estimates, Coefficients of Determination, and the Two Measures of Trading Activity

Beta Estimate	1.000				
Standard Error of Beta Estimate	.517 (.001)	1.000			
Coefficient of Determination	.416 (.001)	-.353 (.001)	1.000		
Mean Trading Level	-.174 (.005)	-.083 (.114)	-.113 (.050)	1.000	
Mean Relative Trading Level	.145 (.074)	.339 (.001)	-.178 (.004)	.504 (.001)	1.000
		S.E.	R^2	Volume	% Volume

p values are provided in parenthesis

Table 2: Standard Errors of Beta Estimates and Coefficients of Determination for Portfolios Comprised of High and Low Trading Activity Securities

Low Trading Activity Portfolios			High Trading Activity Portfolios		
	$SE(\hat{\beta})$	R^2		$SE(\hat{\beta})$	R^2
1.	.048798	.88	1.	.075386	.76
2.	.046835	.89	2.	.067692	.79
3.	.047845	.88	3.	.081502	.82
4.	.049331	.88	4.	.057319	.84
5.	.050584	.87	5.	.072771	.76
6.	.055649	.85	6.	.059295	.83
7.	.053524	.86	7.	.066811	.79
8.	.048879	.88	8.	.091782	.67
9.	.058106	.83	9.	.075568	.75
10.	.039223	.92	10.	.107350	.60
11.	.035729	.93	11.	.068007	.76
12.	.054861	.85	12.	.088218	.69
13.	.050503	.87	13.	.071965	.77
14.	.057546	.84	14.	.064849	.80
15.	.047143	.89	15.	.085449	.70
16.	.045905	.89	16.	.078516	.74
17.	.046708	.89	17.	.089259	.68
18.	.044742	.90	18.	.088000	.69
19.	.047776	.88	19.	.075793	.79
20.	.052563	.86	20.	.066490	.80
21.	.044735	.90	21.	.072865	.76
22.	.043888	.90	22.	.054994	.85
23.	.053089	.86	23.	.081623	.72
24.	.058200	.83	24.	.067601	.79
25.	.046875	.89	25.	.073981	.76
26.	.043971	.90	26.	.070013	.78
27.	.051041	.87	27.	.082179	.72
28.	.045217	.89	28.	.070492	.78
29.	.049705	.88	29.	.091564	.67
30.	.056958	.84	30.	.055748	.85
31.	.056025	.85	31.	.072793	.76
32.	.041914	.91	32.	.075003	.75
33.	.052764	.86	33.	.074641	.75
34.	.058087	.84	34.	.064409	.81

Table 2: Continued.

35.	.057512	.84	35.	.062080	.76
36.	.054080	.85	36.	.085835	.70
37.	.054991	.85	37.	.089136	.68
38.	.058857	.83	38.	.066274	.80
39.	.045143	.89	39.	.054259	.85
40.	.047334	.89	40.	.082177	.72
41.	.044708	.90	41.	.067412	.79
42.	.044759	.90	42.	.056582	.84
43.	.056537	.84	43.	.061714	.82
44.	.046816	.89	44.	.074087	.76
45.	.045466	.89	45.	.073187	.77
46.	.046012	.89	46.	.075086	.75
47.	.052707	.86	47.	.062038	.82
48.	.040479	.91	48.	.104000	.61
49.	.043531	.90	49.	.052470	.86
50.	.047457	.88	50.	.082409	.72
51.	.046980	.89	51.	.075807	.75
52.	.047000	.89	52.	.066935	.79
53.	.044333	.90	53.	.056048	.85
54.	.047871	.88	54.	.084798	.71
55.	.055397	.85	55.	.074306	.76
56.	.042792	.90	56.	.108790	.59
57.	.070571	.78	57.	.076226	.75
58.	.049523	.88	58.	.084271	.71
59.	.052779	.86	59.	.071263	.77
60.	.044911	.90	60.	.068957	.78

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